Diss Cluster

Calculations Policy

Aims and Rationale

The aim of this policy is to create a common, cross phase approach to the teaching of calculations across the Diss cluster of schools. We want our children to be successful, confident with their maths and ready for their transition to high school.

The following cluster schools have been involved in the consultation process:

All Saints Primary School, Winfarthing

Bressingham Primary School

Burston Primary School

Dickleburgh VC Primary School

Diss High School

Diss Infant School

Diss C of E Junior School

Garboldisham Church Primary School

Roydon Primary School

Scole Primary School

St. Andrew's CE VA Primary School, Lopham

Tivetshall Primary School

Rationale

Through collaborative work across the phases, it was decided in the Summer term 2014 that a common approach to teaching calculations was needed. Evidence of the methods that children were using across the cluster was collected. This was analysed, in addition to reference being made to the Norfolk Calculations Research project (2012).

Children should be able to enjoy success when carrying out written methods calculation. This should be underpinned by the effective teaching of mental strategies. The development of conceptual understanding is crucial. Children's learning must be supported by the use of models and images at all ages, fostering a real understanding of the structure of mathematics and why the calculations work.

With the introduction of the New Primary Curriculum in 2014, there is a requirement for pupils to use vertical methods of calculation by the end of Key Stage 2. An opportunity for this has been incorporated to each of the four operations.

Strategies utilised at Diss High School have been clearly identified in the policy. This is to support pupils' transition and their readiness for high school.

Children attending Diss Cluster schools should be enabled to achieve the best possible success in Mathematics.

Introduction

Children are introduced to the processes of calculation through ***practical***, ***oral*** and ***mental*** activities. As children begin to understand the underlying ideas they *develop ways of recording to support their thinking* and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved.

Over time children learn how to *use models and images*, such as empty number lines, to **support their mental and informal written methods of calculation**. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. This firm foundation can be built on further in the progression to high school.

When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. They will do this by asking themselves:

- Can I do this in my head?
- Can I do this in my head using drawing or jottings?
- Do I need to use a pencil and paper procedure?

At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of **number facts**, along with those **mental skills** that are needed to carry out the process and judge if it was successful.

The overall aim is that when children leave primary school they:

- **have a secure knowledge of number facts and a good understanding of the four operations;**
- **are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;**
- **make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;**
- **have an efficient and reliable written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;**
- **be 'high school ready'**

Written methods for addition of whole numbers

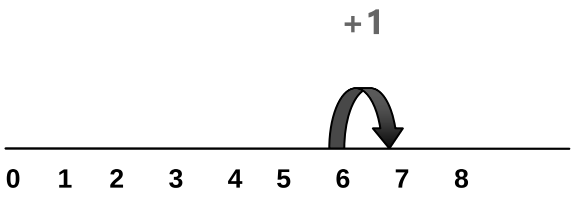
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for addition which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers.

To add successfully, children need to be able to:

- recall all addition pairs to 9 + 9 and complements in 10 e.g. 6 + 4 = 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Note: *It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.*

Resources – Models and images: ITP's, number line, place value cards, hundred square, bead strings etc

Stage 1	
At first children will relate addition to the combining of 2 groups: *Count out 3, count out 2.* *Put together and count out 5* Children are encouraged to make use of fingers as these are a constantly available resource for calculations at this level.	For example: 3 + 2 = 5  Alternatively, count out 3 and then **count on** 2 more to make 5
Stage 2	
The next step is to be able to count ***one more,*** and then several more, on a number line:	For example: 6 + 1 = 7 

Stage 3

Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

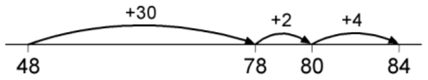
The mental methods involve partitioning, e.g. adding the tens and ones separately, often starting with the tens.

The empty number line helps to record the steps on the way to calculating the total.

This method can be used for the addition of larger numbers and decimals further up the school.

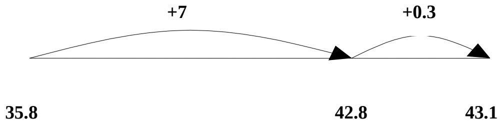
8 + 7 = 15

48 + 36 = 84



or:

35.8 + 7.3 = 43.1



Stage 4

The next stage is to record mental methods using partitioning.

Add the tens and then the ones to form partial sums and then add these partial sums.

Move on to a layout showing the addition of the tens to the tens and the ones to the ones separately. To find the partial sums either the tens or the ones can be added first, and the total of the partial sums can be found by adding them in any order.

*The addition of the tens in the calculation 47 + 76 is described in the words '**forty plus seventy equals one hundred and ten'**, stressing the link to the related fact 'four plus seven equals eleven'*

Record steps in addition using partitioning:

47 + 76 = 40 + 70 = 110
7 + 6 = 13
123

Three digit numbers:

187	or	187	or	187
246		274		274
300		300		13
120		120		120
13		13		300
400		433		433
30				
3				
433				

Decimals

14.28
17.56
20.00
11.00
0.70
0.14
31.84

Stage 5

Carrying

Pencil and paper procedures

In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.

Later, extend to adding three two digit numbers, two three digit numbers and numbers with different numbers of digits.

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 123 \\ \hline 11 \end{array} \qquad \begin{array}{r} 258 \\ +\ 87 \\ \hline 345 \\ \hline 11 \end{array} \qquad \begin{array}{r} 366 \\ +458 \\ \hline 824 \\ \hline 11 \end{array}$$

8 tenths add 6 tenths makes 14 tenths, or 1 whole and 4 tenths. The 1 whole is 'carried' into the units column and the 4 tenths is written in the tenths column.

$$\begin{array}{r} 72.8 \\ +\ \ 54.6 \\ \hline 127.4 \\ \hline 1 \end{array}$$

Extend to numbers with any number of digits and decimals with 1 and 2 decimal places**.**

$$\begin{array}{r} 13.86 \\ +\ \ 9.481 \\ \hline 23.341 \\ \hline 1\,11 \end{array}$$

*Before they use a written method to add decimal numbers, children should **estimate the answer**.*

For example, they calculate 13.86 + 9.481, and use rounding to check that their answer is approximately 23, rounding to check that their answer is approximately 23

This is the 'high school' ready strategy

Written methods for subtraction of whole numbers

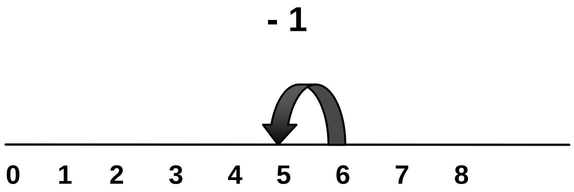
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for subtraction which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers .

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Note: *It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.*

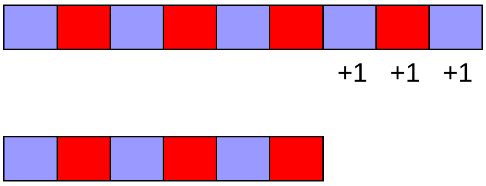
Stage 1	
In the early stages, children will be taught to **'take away'** one or two objects and find the new total.	For example: 5-3=2  **5 take 2 away is 3** 
Stage 2	
The next stage is for children to be able to work out **one less** or several less on a number line	For example: 6 – 1 = 5 

Stage 3

At an early stage children are introduced to the concept of difference and that subtraction can be worked out by counting on the difference.

For example: How much longer is this row of cubes than this one?

9 – 3 = 6



Stage 4

Using the empty number line

The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.

*(The steps can also be recorded by **counting up** from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.)*

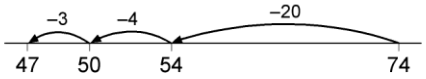
With practice, children will need to record less information and decide whether to count back or forward.

It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.

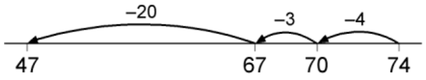
Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

15 – 7 = 8

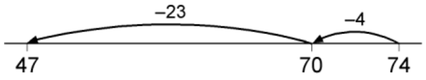
74 – 27 = 47 worked by counting back:



The steps may be recorded in a different order:



or combined:

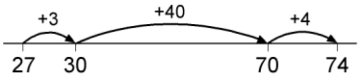


Stage 5

The counting up method

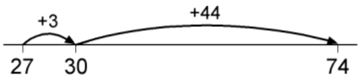
The mental method of **counting up** from the smaller to the larger number can be recorded using either number lines or vertically in columns.

The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ☐ = 74 mentally.



$$\begin{array}{rl} 74 & \\ -\ \underline{27} & \\ 3 & \rightarrow 30 \\ 40 & \rightarrow 70 \\ \underline{4} & \rightarrow 74 \\ 47 & \end{array}$$

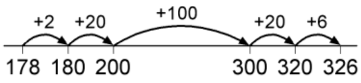
or:



$$\begin{array}{rl} 74 & \\ -\ \underline{27} & \\ 3 & \rightarrow 30 \\ \underline{44} & \rightarrow 74 \\ 47 & \end{array}$$

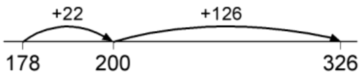
With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ☐ = 200 and 200 + ☐ = 326 mentally.

The most compact form of recording remains reasonably efficient.



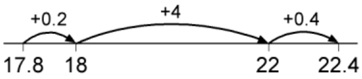
$$\begin{array}{rl} 326 & \\ -\underline{178} & \\ 2 & \rightarrow 180 \\ 20 & \rightarrow 200 \\ 100 & \rightarrow 300 \\ \underline{26} & \rightarrow 326 \\ 148 & \end{array}$$

or:



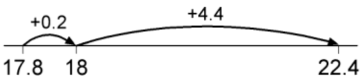
$$\begin{array}{rl} 326 & \\ -\underline{178} & \\ 22 & \rightarrow 200 \\ \underline{126} & \rightarrow 326 \\ 148 & \end{array}$$

- The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed.
- This counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.



$$\begin{array}{rl} 22.4 & \\ -\underline{17.8} & \\ 0.2 & \rightarrow 18 \\ 4.0 & \rightarrow 22 \\ \underline{0.4} & \rightarrow 22.4 \\ 4.6 & \end{array}$$

or:

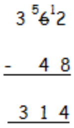


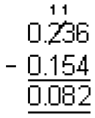
$$\begin{array}{rl} 22.4 & \\ -\underline{17.8} & \\ 0.2 & \rightarrow 18 \\ \underline{4.4} & \rightarrow 22.4 \\ 4.6 & \end{array}$$

Stage 6

Decomposition should only be taught when children are secure in their understanding and have a strong awareness of place value.

This is the ‘high school’ ready strategy





Written methods for multiplication of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication, two-digit by two-digit multiplication, and three-digit by two-digit multiplication.

To multiply successfully, children need to be able to:

- recall all multiplication facts to 10 × 10;
- partition number into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers

Note: *It is important that children’s mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication*

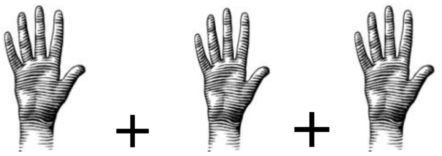
Stage 1

First children are taught to count in 2’s, 10’s and 5’s using practical objects.

For example:

2 x 3 = 6

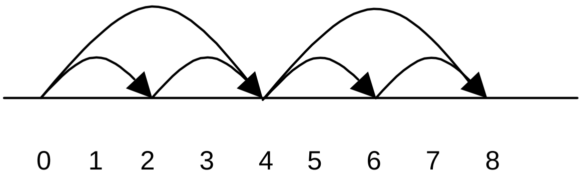
5 x 3 = 15



= 15

Stage 2

Number Line Jumps

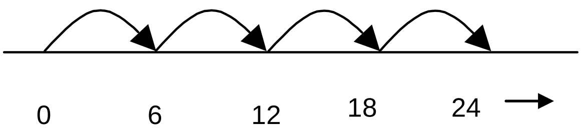


4 + 4 = 8

2 x 4 = 8

Or 4 x 2 = 2 + 2 + 2 + 2 + 2

Number line jumps support mental methods

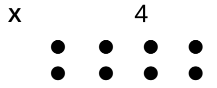


13 x 6 = 78

Stage 3:

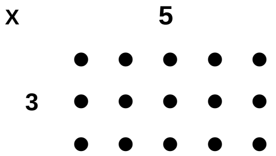
Arrays

Arrays & Repeated addition



4 x 2 = 2 + 2+ 2 + 2 = 8

2 x 4 = 4 + 4 = 8



5 x 3 =15

3 x 5 = 15

15 ÷ 3 = 5

15 ÷ 5 = 3

Stage 4

Partitioning units and tens, hundreds or thousands

Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.

43 x 6 = 40 x 6 = 240

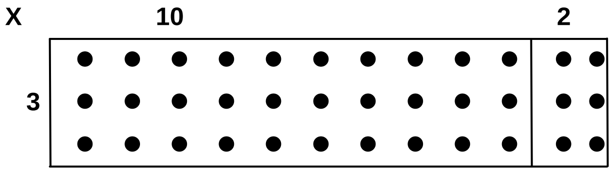
3 x 6 = 18

258

Stage 5

Arrays to grid

Arrays to grid 12 x 3



x	10	2
3	30	6

= 30 + 6 = 36

Stage 6

The grid method

As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.

- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266

x	30	8
7	210	56

210 + 56 =200 + 60 + 6 = 266

Or
210
56
200
60
6
266

Stage 7

Two-digit by two-digit products

Extend to **TU × TU**, asking children to estimate first.

Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.

56 × 27 is approximately 60 × 30 = 1800.

x	20	7
50	1000	350
6	120	42

1000
350
120
42
1000
400
110
2
1512

Stage 8

Three-digit by two-digit products

X	20	9
200	4000	1800
80	1600	720
6	120	54

$$\begin{array}{r} 4000\\ 1800\\ 1600\\ 720\\ 120\\ \underline{\quad 54}\\ 6000\\ 2200\\ 90\\ \underline{\quad 4}\\ 8294 \end{array}$$

Stage 9

Vertical method

Only consider teaching when children are secure with the grid method above and have a good understanding of place value.

56 × 27 is approximately 60 × 30 = 1800.

$$\begin{array}{rr} 56 & \\ \times\ \underline{\ 27} & \\ 1000 & 50\times20=1000\\ 120 & 6\times20=\ \ 120\\ 350 & 50\times7=\ \ 350\\ \underline{\ 42} & 6\times7=\ \ \ 42\\ \underline{1512} & \\ 1\ \ & \end{array}$$

Or

$$\begin{array}{rr} 56 & \\ \underline{x\ \ 27} & \\ 42 & 7\ x\ \ 6\ =\ \ \ 42\\ 350 & 7\ x\ 50 =\ \ 350\\ 120 & 20\ x\ \ 6 =\ \ 120\\ \underline{1000} & 20\ x\ 50 = 1000\\ \underline{1512} & \\ 1\ \ \ & \end{array}$$

Stage 10

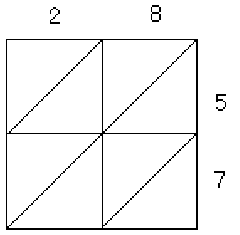
Lattice Method

Children will need to be organised with setting out their work and be able to follow the steps accurately.

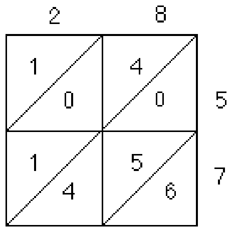
This is the ‘high school’ ready strategy

28 × 57 =

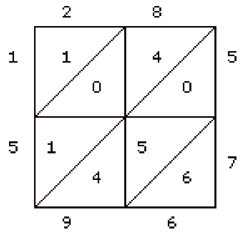
As 28 and 57 have two digits each, a lattice is set out with two columns and two rows. The diagonals are drawn in each cell as shown below. 28 is written above the lattice with 2 above the first column and 8 above the second. 57 is written to the right of the lattice with 5 along the first row and 7 along the second.



The partial products of these digits taken two at a time is set out in the corresponding cells with the tens above the diagonal and ones below. For example, the partial products in this case are 5 × 8 (= 40), 5 × 2 (= 10), 7 × 8 (= 56) and 7 × 2 (= 14).These are set out as shown below.

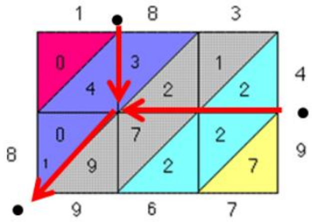


The sum along each diagonal is then recorded as shown below and these digits 1, 5, 9 and 6 form the answer to the multiplication. As usual, start adding at the ones (in this case ‘6’ which comes from multiplying 8 ones by 7 ones), proceeding from right to left around the lattice.



28 × 57 = 1596

This method can also be applied to calculations with decimals: 1.83 x 4.9 =



1.83 x 4.9 = 8.967

Written methods for division of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for division which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to long division through upper Key Stage 2 – first long division TU ÷ U, extending to HTU ÷ U and then HTU ÷ TU.

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

Note: *It is important that children’s mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.*

To carry out written methods of division successful, children also need to be able to:

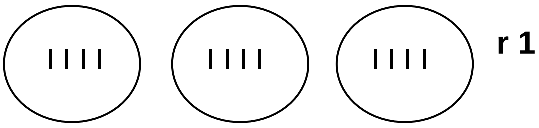
- understand division as repeated addition;
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- add numbers using an appropriate method.

Stage 1

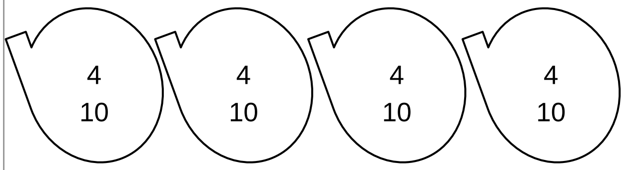
Understanding division as <u>sharing</u>

6 cakes are shared between 2 people. How many cakes do they have each?

6 ÷ 2 = 3



13 ÷ 3 = 4 r 1

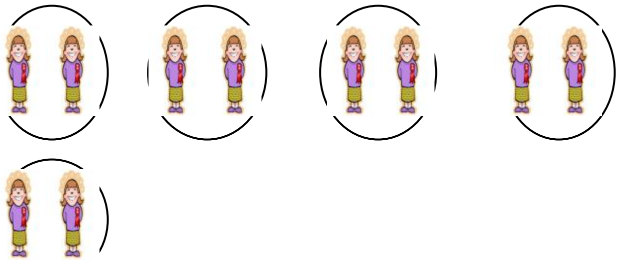


58 ÷ 4 = 14 r 2

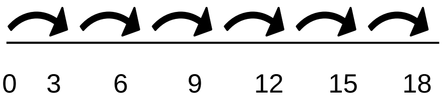
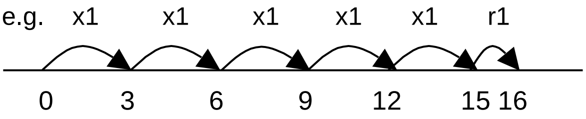
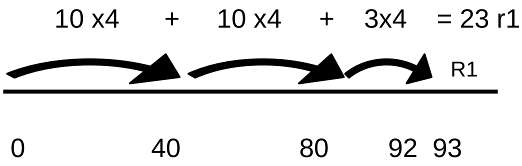
Stage 2

Understanding division as <u>grouping</u>

10 children are grouped into teams of 2. How many teams will there be?



10 ÷ 2 = 5

Stage 3	
Counting on a number line A number line can be used to record grouping or sharing. It is also possible to record counting down in this way.	For example: 18 ÷ 3 = 6.  T ɔe shared between three children OR 18 chocolate eggs that are to be packed (grouped) in boxes of 3 **Remainders** (≈ Year 3) 16 ÷ 3 = 5 r1 **Grouping – How many 3's make 16, how many left over?** Sharing - 16 shared between 3, how many left over? e.g. 
This method supports both mental division strategies and the expanded strategy shown below. Children can extend to multiples or 'chunks' of the divisor eg 93 ÷ 4 = (40 ÷ 4) + (40 ÷ 4) + (12 ÷ 4) + (1 ÷ 4) = 23 ¼ Remainder 1 should also be seen as 1 out of a possible four and therefore 1/4 .	93 ÷ 4 

Stage 4

Climbing/chunking up method

For TU ÷ U there is a link to the mental method.

As you record the division, ask:

'How many sevens make seventy?', 'How many sevens in 70?' or 'What is 70 divided by 7?'

97 ÷ 7

10 x 7 = 70
3 x 7 = 21
13 91
Answer: 13 R6

$$\begin{array}{r l} 9\overline{)97} & \\ -\underline{90} & 9\times 10 \\ 7 & \end{array}$$

Answer: 10 R 7

Once they understand and can apply the method, children should be able to move on from TU ÷ U to **HTU ÷ U** quite quickly as the principles are the same.

- This method, often referred to as 'chunking', is based on adding multiples of the divisor, or 'chunks'. Initially children add several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to add.
- Chunking is useful for reminding children of the link between division and repeated addition.

Chunking up avoids any difficulties that children have with subtraction.

- However, children need to recognise that chunking is inefficient if too many additions or subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.

257 ÷ 8

$$\begin{array}{r r} 80 & \text{x10} \\ +\underline{\ 80} & \text{x10} \\ 160 & \\ +\underline{\ \ 80} & \text{x10} \\ 240 & \\ +\underline{\ \ 16} & \underline{\text{x2}} \\ 256 & 32 \end{array}$$

Answer: 32R1

Stage 5

Chunking up using known facts

The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that 'trap' the HTU dividend.

Estimating has two purposes when doing a division:

- to help to choose a starting point for the division;
- to check the answer after the calculation.

Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.

To find 196 ÷ 6, we start by multiplying 6 by 10, 20, 30, … to find that **30 x 6 = 180** and 40 x 6 = 240. The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40. The calculation is then completed by any of the strategies demonstrated above.

196 ÷ 6

10 x 6 = 60
20 x 6 = 120
30 x 6 = 180
40 x 6 = 240

30 x 6 = 180 180 + 16 = 196
2 x 6 = 12 180 + 12 = 192
32
196 ÷ 6 = 32R4

The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.

This calculations could be completed by chunking down.

Stage 6	
Short Division **This is the 'high school' ready strategy**	For 81 ÷ 3, the dividend of 81 is split into 60, the highest multiple of 3 that is also a multiple 10 and less than 81, to give 60 + 21. Each number is then divided by 3. The short division method is recorded like this: $\begin{array}{r} 2\,7 \\ 3\overline{)8\,{}^{2}1} \end{array}$ For 291 ÷ 3, because 3 × 90 = 270 and 3 × 100 = 300, we use 270 and split the dividend of 291 into 270 + 21. Each part is then divided by 3. The short division method is recorded like this: $\begin{array}{r} 9\,7 \\ 3\overline{)2\,9\,{}^{2}1} \end{array}$

Stage 7	
Long Division **This is the 'high school' ready strategy** The next step is to tackle HTU ÷ TU, which for most children will be in upper Key Stage 2. • The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left (where 20 and 3 are written below the 24) of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient.	How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560. $\begin{array}{rrl} & 24\overline{)\,560} & \\ 20 & -\underline{480} & 24\times20 \\ & 80 & \\ 3 & \underline{72} & 24\times3 \\ & 8 & \end{array}$ Answer: 23 R 8 This calculation could be completed by chunking up.